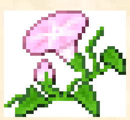
7．5 逆行列

$$AB = I \qquad B = A^{-1}$$

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} |A_{11}| & |A_{21}| & \cdots & |A_{n1}| \\ |A_{12}| & |A_{22}| & \cdots & |A_{n2}| \\ \vdots & \vdots & & \vdots \\ |A_{1n}| & |A_{2n}| & \cdots & |A_{nn}| \end{pmatrix}$$

$|A_{ji}| \Rightarrow A$のij余因子



７．５．２　ガウス･ジョルダン法

変形する

逆行列

$$
\left[\begin{array}{cccc|cccc}
1 & 0 & \cdots & 0 & a_{11}^{-1} & a_{12}^{-1} & \cdots & a_{1n}^{-1} \\
0 & 1 & \cdots & 0 & a_{21}^{-1} & a_{22}^{-1} & \cdots & a_{2n}^{-1} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
0 & 0 & \cdots & 1 & a_{n1}^{-1} & a_{n2}^{-1} & \cdots & a_{nn}^{-1}
\end{array}\right]
$$

例８　次の逆行列をガウス･ジョルダン法で求めよ

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 1 & -2 \\ 1 & 5 & 1 \end{bmatrix}$$

拡大行列

$$\left[\begin{array}{ccc|ccc} 2 & -1 & 3 & 1 & 0 & 0 \\ -1 & 1 & -2 & 0 & 1 & 0 \\ 1 & 5 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 2 & -1 & 3 & 1 & 0 & 0 \\ -1 & 1 & -2 & 0 & 1 & 0 \\ 1 & 5 & 1 & 0 & 0 & 1 \end{array}\right]$$

2で割る

$$\left[\begin{array}{ccc|ccc} 1 & -1/2 & 3/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & -1/2 & 1/2 & 1 & 0 \\ 0 & 11/2 & -1/2 & -1/2 & 0 & 1 \end{array}\right]$$

1行目を足す

1行目を引く

$$\left[\begin{array}{ccc|ccc} 1 & -1/2 & 3/2 & 1/2 & 0 & 0 \\ 0 & 1 & -1 & 1 & 2 & 0 \\ 0 & 11/2 & -1/2 & -1/2 & 0 & 1 \end{array}\right]$$

2倍する

$$\left[\begin{array}{ccc|ccc} 1 & \color{red}{0} & \color{red}{1} & \color{red}{1} & \color{red}{1} & 0 \\ 0 & 1 & -1 & 1 & 2 & 0 \\ 0 & 0 & 5 & -6 & -11 & 1 \end{array}\right]$$

3行目で引く

11／2

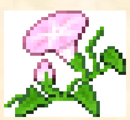
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 11/5 & 16/5 & -1/5 \\ 0 & 1 & 0 & -1/5 & -1/5 & 1/5 \\ 0 & 0 & 1 & -6/5 & -11/5 & 1/5 \end{array}\right]$$

3行目を足す

5で割る

$$A^{-1} = \begin{bmatrix} 11/5 & 16/5 & -1/5 \\ -1/5 & -1/5 & 1/5 \\ -6/5 & -11/5 & 1/5 \end{bmatrix}$$

逆行列



7．5．3　LU分解法による逆行列

$$A = LU$$

三角分解

$$\boldsymbol{A}^{-1} = \left(\boldsymbol{L}\boldsymbol{U}\right)^{-1}$$

$$\boldsymbol{A}^{-1} = \boldsymbol{U}^{-1}\boldsymbol{L}^{-1}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} u_{11}^{-1} & u_{12}^{-1} & u_{13}^{-1} \\ 0 & u_{22}^{-1} & u_{23}^{-1} \\ 0 & 0 & u_{33}^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$u_{11}u_{11}^{-1} = 1 \qquad u_{22}u_{22}^{-1} = 1 \qquad u_{33}u_{33}^{-1} = 1$$

$$u_{11}^{-1} = 1/u_{11} \qquad u_{22}^{-1} = 1/u_{22} \qquad u_{33}^{-1} = 1/u_{33}$$

$$u_{11}u_{12}^{-1} + u_{12}u_{22}^{-1} = 0 \qquad u_{12}^{-1} = -u_{12}u_{22}^{-1}/u_{11}$$

$$u_{22}u_{23}^{-1} + u_{23}u_{33}^{-1} = 0 \qquad u_{23}^{-1} = -u_{23}u_{33}^{-1}/u_{22}$$

$$u_{11}u_{13}^{-1} + u_{12}u_{23}^{-1} + u_{13}u_{33}^{-1} = 0$$

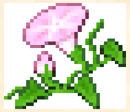
$$u_{13}^{-1} = -\left(u_{12}u_{23}^{-1} + u_{13}u_{33}^{-1}\right)/u_{11}$$

例９　右のＵ行列の逆行列を求めよ

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_{11}^{-1} & \boldsymbol{u}_{12}^{-1} & \boldsymbol{u}_{13}^{-1} \\ 0 & \boldsymbol{u}_{22}^{-1} & \boldsymbol{u}_{23}^{-1} \\ 0 & 0 & \boldsymbol{u}_{33}^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boldsymbol{u}_{11}^{-1} = 1$　　$\boldsymbol{u}_{22}^{-1} = 1/4$　　$\boldsymbol{u}_{33}^{-1} = 1/6$



$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} u_{11}^{-1} & u_{12}^{-1} & u_{13}^{-1} \\ 0 & u_{22}^{-1} & u_{23}^{-1} \\ 0 & 0 & u_{33}^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$u_{11}^{-1} = 1$ $\qquad u_{22}^{-1} = 1/4$ $\qquad u_{33}^{-1} = 1/6$

$$1 \times u_{12}^{-1} + 2 \times \frac{1}{4} = 0 \qquad u_{12}^{-1} = -1/2$$

$$4 \times u_{23}^{-1} + 5 \times \frac{1}{6} = 0 \qquad u_{23}^{-1} = -5/24$$

$$u_{13}^{-1} + 2 \times \left(-\frac{5}{24}\right) + 3 \times \frac{1}{6} = 0 \qquad u_{13}^{-1} = -1/12$$